Uncertainty on survival probabilities and solvency capital requirement : application to long-term care insurance

- Frédéric PLANCHET (Université Lyon 1, Laboratoire SAF)
- Julien TOMAS (Université Lyon 1, Laboratoire SAF)
Summary

In this paper, we focus on uncertainty issues on survival probabilities of LTC-claimants and its consequences on solvency capital requirement. Among the risks affecting long-term care portfolios, special attention is addressed to the table risk, i.e. the risk of unanticipated aggregate mortality, arising from the uncertainty in modeling LTC-claimants survival law. The table risk can be thought as the risk of systematic deviations referring not only to a parameter risk but, as well, to any other sources leading to a misinterpretation of the life table resulting for example from an evolution of medical techniques or a change in rules of acceptance. In fine, the idea is to introduce the risk of systematic deviations arising from the uncertainty on the conditional probability of death directly. We analyze the consequences of an error of appreciation on the LTC-claimants survival probabilities in terms of level of reserves and describe a framework in an Own Risk and Solvency Assessment perspective to measure the gap between the risk profile from the standard formula to the risk analysis specific to the organism.

Keywords. Own Risk and Solvency Assessment, Solvency Capital Requirement, Long-term care insurance, Risk of systematic deviations, Table risk, Semi-parametric model, Proportional hazard, Frailty.

Résumé

Dans cet article, on s’intéresse à un modèle permettant de prendre en compte l’incertitude sur la loi de survie d’individus dépendants et les conséquences sur le besoin en fonds propres dans le cadre du dispositif Solvabilité II. Parmi les risques affectant les portefeuilles d’assurance dépendance, une attention particulière est adressée au risque de table, à savoir le risque de mortalité totale imprévue résultant de l’incertitude dans la modélisation de la loi de maintien en dépendance. Le risque de table peut être considéré comme le risque de déviations systématiques. Il intègre le risque d’estimation mais aussi a priori d’autres sources potentielles de méconnaissance de la table résultant par exemple d’une évolution des techniques médicales ou des règles d’acceptation. In fine, l’idée est d’introduire le risque systématique associé à l’aléa sur les probabilités conditionnelles de décès directement. On analyse les conséquences d’une erreur d’appréciation sur les durées de maintien en termes de niveau de provisions et on décrit un cadre utilisable dans une logique ORSA pour mesurer l’écart entre le profil de risque issu de la formule standard et celui issu d’une analyse des risques spécifiques à l’entité.

Mots-clés. ORSA, SCR, Assurance dépendance, Risque de déviations systématiques, Risque de table, Modèle semi-paramétrique, Hasard proportionnel, Modèle de fragilité.
1 Introduction

The important change in Solvency II is the general reform of assessment of the insurer solvency. The solvency is not only a matter of equity, but a meaningful process of accountability in which the insurer must monitor its activities through an adapted system of management and risk control. The ORSA (Own Risk and Solvency Assessment) carries the key issues of the reform of Solvency II. It is the set of processes that contribute to the regular assessment of the overall internal solvency of the company as an integral part of the business strategy and taking into account the specific risk profile of the insurer. This identification of the specific risk profile of the company is the cornerstone of an effective governance. The ORSA allows to shift from a logic of retrospective risks control to a logic of steering by monitoring the risks which incorporates the solvency. By identifying factors that may affect the current and future solvency, the ORSA offers the opportunity to the insurer to respond promptly and effectively to the economic context.

The need for a sound assessment of a LTC (Long-Term Care) insurer’s risk profile suggests a comprehensive approach to the valuations of this particular life insurance business. LTC is a mix of social and health care provided on a daily basis, formally or informally, at home or in institutions, to people suffering from a loss of mobility and autonomy in their activity of daily living. Although loss of autonomy may occur at any age, its frequency rises with age. LTC insurance contracts are individual or collective and guarantee the payment of a fixed allowance, in the form of monthly cash benefit, possibly proportional to the degree of dependency, see Kessler (2008) and Courbage and Roudaut (2011) for studies on the French LTC insurance market.

Most of the actuarial publications on this topic focus on the construction of models of projected benefits, see Gauzère et al. (1999) and Deléglise et al. (2009), the assessment of transition probabilities to model the life-history of LTC-patients, see Czado and Rudolph (2002) and Helms et al. (2005), or the construction of the survival distribution of LTC insurance policyholders, see Tomas and Planchet (2012).
The pricing and reserving as well as the management of LTC portfolios are very sensitive to the choice of the mortality table adopted. In addition, the construction of such table is a difficult exercise, see Tomas and Planchet (2012).

In this article, we analyze the consequences of an error of appreciation of the survival probabilities of LTC-claimants in terms of level of reserves. We describe a framework, in an ORSA perspective, to measure the gap between the risk profile from the standard formula to a risk analysis specific to the insurer. In this context, the simplicity of implementation is privileged.

The article is organized as follow. Section 2 has still an introductory purpose and makes precise the notation used in the following. In Section 3, the risk of random fluctuations is briefly sketched. The table risk and an application to the computation of the SCR (Solvency Capital Requirement) are then addressed. Section 4 presents the numerical application. Finally, some remarks in Section 5 conclude the paper.

2 Notation and reserves valuation

2.1 Notation

We consider a LTC insured population of \( n \) individuals with the same level of severity (heavy claimant) with durations of care \( u_i \) (in months), and ages of occurrence of the pathology \( v_i \) with \( i = 1, \ldots, n \). There are two temporal dimensions \( u \) and \( v \). However, they do not have the same status: \( v \) is a variable denoting the heterogeneity while \( u \) represents the variable linked with the duration.

We note \( \Lambda \) the random amount of the valuation of the insurer. It is the sum of the future cash flows discounted with monthly rate \( r \). The total cash flows payable at month \( t \) are denoted \( F_t \). It is the sum of the \( n \) individual care \( c_i \) delivered in the period over the month \( t \). The remaining lifetime of an individual \( i \) when the pathology occurred at age \( v_i \) with the duration of the care \( u_i \) is \( T_{u_i}(v_i) \).

2.2 Reserves valuation

With the notation of Section 2.1, the total cash flows payable at month \( t \) has the form

\[
F_t = \sum_{i=1}^{n} c_i I\{t;+\infty\}(T_{u_i}(v_i))
\]

The sum of total discounted cash flows is then

\[
\Lambda = \sum_{t=1}^{+\infty} F_t (1 + r)^{-t} = \sum_{t=1}^{+\infty} (1 + r)^{-t} \sum_{i=1}^{n} c_i I\{t;+\infty\}(T_{u_i}(v_i))
\]

\[
= \sum_{i=1}^{n} c_i \sum_{t=1}^{+\infty} \frac{I\{t;+\infty\}(T_{u_i}(v_i))}{(1 + r)^t} = \sum_{i=1}^{n} c_i \Psi_i.
\]

In the following, we are interested in the expectation of \( \Lambda \), i.e. the reserve, and more particularly to its law. The reserve has a simple form:

\[
\mathbb{E}[\Lambda] = \sum_{t=1}^{+\infty} (1 + r)^{-t} \sum_{i=1}^{n} c_i \mathbb{P}[T_{u_i}(v_i) > t].
\]

Often, the reserve is written as

\[
\mathbb{E}[\Lambda] = \sum_{i=1}^{n} c_i \pi_i \quad \text{with} \quad \pi = \sum_{t=1}^{+\infty} (1 + r)^{-t} \mathbb{P}[T_u > t].
\]
The parameter $\pi$ is called the coefficient of reserve. For a discount rate $r = 0$, the coefficient of reserve is equivalent to the expectation of remaining lifetime:

$$\pi(u, v) = \sum_{t=1}^{+\infty} P[T_u(v) > t] = E[T_u(v)].$$

When provisioning the amount $E[\Lambda]$, the insurer faces adverse deviations due to two distinct factors:

i. The random fluctuations of the observed mortality rates around the relevant expected values, i.e. the adjusted mortality rates, consequences of the finite size of the population exposed to the risk. The risk of random fluctuations (often called process risk) is diversifiable (one should better said pooling). Its financial impact decreases, in relative terms, as the portfolio size increases.

ii. The inaccuracy of the underlying survival law, from which the probability $P[T_{ui}(vi) > t]$ are derived, is called the table risk. It is the risk of unanticipated aggregate mortality, arising from the uncertainty in modeling LTC-claimants survival law. The table risk can be thought as the risk of systematic deviations referring not only to a parameter risk but, as well, to any other sources leading to a misinterpretation of the life table resulting for example from an evolution of medical techniques or a change in rules of acceptance. The risk of systematic deviations cannot be hedged by increasing the portfolio size. Actually, in relative terms, its severity does not reduce as the portfolio size increases, since deviations concern all the insureds in the same direction.

In the following, the risk of random fluctuations is briefly sketched. The table risk and its impact of the SCR are then addressed.

3 The risk components and SCR

3.1 Risk of random fluctuations

To incorporate the uncertainty arising from the random fluctuations of the observed mortality rates around the relevant expected values, i.e. the adjusted mortality rates, we can construct a confidence interval of sum of the total discounted cash flows around its expected value. Due to the assumption of independence between individuals and as we can reasonably assume that the individual cash flows $\Psi_i$ are bounded by a constant, i.e. that the set $\{\Psi_i\}_{i=1}^n$ is uniformly bounded, the limit distribution of $\Lambda$ is gaussian:

$$\frac{\Lambda - E[\Lambda]}{\sigma_\Lambda} \xrightarrow{n \to +\infty} \mathcal{N}(0, 1),$$

where $\sigma_\Lambda$ denoting the standard deviation of $\Lambda$. Hence, we can approximate the distribution of the total discounted cash flows and we can easily derive quantiles and confidence interval. For example, $\Lambda$ falls in the random interval with approximately $(1 - \alpha)$ coverage probability,

$$\mathcal{I}_\Lambda = E[\Lambda] \pm \varphi_\alpha \times \sigma_\Lambda,$$

with $\varphi_\alpha$ chosen as the $(1 - \alpha/2)$ quantile of the standard normal distribution.

To assess the two first moments of $\Lambda$, since

$$E[\Lambda] = \sum_{i=1}^n c_i E[\Psi_i] \quad \text{and} \quad V[\Lambda] = \sum_{i=1}^n c_i^2 V[\Psi_i], \quad (1)$$
we only need to know the expectation and variance of

\[ \Psi = \sum_{t=1}^{+\infty} \frac{I\{T_u(v)\}}{(1+r)^t}. \]

One easily finds:

\[ \mathbb{E}[\Psi^2] = \sum_{t=1}^{+\infty} \frac{S_v(t)}{(1+r)^{2t}} + 2 \times \sum_{t=2}^{+\infty} \frac{1}{r} \left( \frac{1}{(1+r)^t} - \frac{1}{(1+r)^{2t-1}} \right) S_v(t), \]

where \( S_v(t) \) denotes the survival function when pathology occurred at age \( v \) for duration \( t \). However, it is simpler to start with a continuous expression of \( \Psi \),

\[ \Psi = \int_0^{+\infty} \exp(-\tau t) \frac{I\{T_u(v)\}}{dt} \quad \text{with} \quad \tau = \ln(1+r). \]

It leads to

\[ \mathbb{E}[\Psi] = \int_0^{+\infty} \exp(-\tau t) S_v(t) dt \approx \sum_{t\geq 1} \exp(-\tau t) S_v(t), \quad (2) \]

and

\[ \mathbb{E}[\Psi^2] = \frac{2}{\tau} \int_0^{+\infty} \left( 1 - \exp(-\tau t) \right) \exp(-\tau t) S_v(t) dt \approx \frac{2}{\tau} \sum_{t\geq 1} \left( 1 - \exp(-\tau t) \right) \exp(-\tau t) S_v(t). \quad (3) \]

Note that the discrete approximation of the expectation is identical to the one obtained previously, while the approximation of the variance differs slightly.

We will not go further in assessing the risk of random fluctuations. Extensive studies have discussed the issue. The variance and covariance of parameter estimates are derived either by standard estimation or by bootstrapping, resampling from the original data to create replicate datasets from which the variance and covariance can be estimated. See Planchet and Kamega (2013) for an application of parametric bootstrap. In the following we focus on the table risk arising from the uncertainty in modeling LTC-claimants survival law.

### 3.2 Table risk

The choice of the mortality table adopted has a crucial impact on the pricing and reserving as well as the management of LTC portfolios. Tomas and Planchet (2012) have shown that the construction of such table is a difficult exercise. Unlike the risk of random fluctuations, the table risk is systematic, due to the fact that it concerns aggregate mortality. It is realized when deviations from expected mortality are observed along the duration of the care. Here, the idea is to introduce the risk of systematic deviations arising from the uncertainty on the conditional probability of death directly with a semi-parametric approach.

The simplest way to introduce an uncertainty on the expected mortality is to add a disturbance on the logits of the adjusted probabilities of death, see Planchet and Théron (2011). We note the adjusted conditional probability of death at duration \( u \) for the age of occurrence \( v \) by \( q_u(v) \).

Then,

\[ \text{logit} \ q_u^0(v) = \ln \left( \frac{q_u(v)}{1-q_u(v)} \right) = \ln \left( \frac{q_u(v)}{1-q_u(v)} \right) + \epsilon, \quad (4) \]

where \( \epsilon \) is a variable centered which we suppose to be gaussian in the following.
Equivalently,
\[ q^a_u(v) = \frac{a \times \exp(\logit q_a(v))}{1 + a \times \exp(\logit q_a(v))}, \quad \text{with} \quad a = \ln \epsilon. \tag{5} \]

The disturbance is controlled by the volatility of \( \epsilon \), noted \( \sigma_\epsilon \). We will vary \( \sigma_\epsilon \) from 1 to 20 \%, and measure
the uncertainty on the expectation of the remaining lifetime in computing the relative difference, denoted \( \delta \), between the expectation and the 95 \% quantile of the simulated remaining lifetime:
\[ \delta = \frac{\rho_{95\%}(E[T|a]) - E[E[T|a]]}{E[E[T|a]]} = \frac{\rho_{95\%}(E[T|a]) - E[T]}{E[T]}. \tag{6} \]

We can also specify model (5) through a parametric perspective in order to have explicit expressions
for the survival law. In a continuous context, we suppose that a hazard function with a known parameter \( \theta \) can describe model (5). The relation (4) is then expressed through
\[ \ln h^\theta_{\theta | a} = \ln h^\theta_{\theta} + \epsilon \quad \text{or equivalently} \quad h^\theta_{\theta | a} = a \times h^\theta. \]

The previous relation falls in the framework of proportional hazard models and
\[ E[T^\theta_{\theta | a}] = \int_0^{+\infty} S^\theta_{\theta}(t) \, dt. \]

This class of models refers to the frailty models introduced in Vaupel et al. (1979) used in a mortality
context to express unobserved heterogeneity. The authors assume the frailty at birth to be gamma dis-
tributed, while in our context, the log-normal distribution would be more appropriate.

We only need to specify the form of the hazard function to have a fully parametric model. For instance
in a Makeham model, Makeham (1867) and Makeham (1890), \( h^\theta_{\theta} = \alpha + \beta \times \gamma^t \) and the survival function
writes
\[ S^\theta_{\theta}(t) = \exp\left(-\alpha t - \frac{\beta}{\ln \gamma} (\gamma^t - 1)\right), \]
and in particular
\[ E[T^\theta] \approx \sum_{t>0} \exp\left(-\alpha t - \frac{\beta}{\ln \gamma} (\gamma^t - 1)\right). \]

The disturbed model is a Makeham model as well. It leads to the following explicit expression for the
conditional survival law:
\[ E[T^\theta_{\theta | a}] = \sum_{t>0} \exp\left(-a \left(-\alpha t - \frac{\beta}{\ln \gamma} (\gamma^t - 1)\right)\right). \]

3.3 Application to the SCR

The current standard requirements for the Solvency II life risk module have been specified in QIS5,
CEIOPS (2010, pp.147-163). QIS5 prescribes a SCR which accounts explicitly for the uncertainty arising
from the systematic deviations and parameters estimation but not for the random fluctuations and process
risk (severity of claims). In fact, the process risk has been disregarded as not significant enough, and has
been included in the systematic and parameter risk component, in order to simplify the standard formula.
The severity of the risk of random fluctuation decreases, in relative terms, as the portfolio size increases.
Hence, we can suppose, for sufficient exposure, that the risk of systematic deviations and parameter risk
have a larger financial impact. Here, our aim is then to measure the relevancy of the shocks described in
the QIS5 specification with the specific risks supported by the insurer in an ORSA perspective.
The expression (6) provides information about the distribution of the total sum of the discounted cash flows, $\Lambda$. However, computing the quantiles of this distribution gives a biased evaluation of the SCR, because it does not take into account the limitation of the projection (computed to infinity), and the risk margin. Following Guibert et al. (2010, Section 3), we use the general approximation

$$
\text{SCR} \approx \frac{\text{VaR}_{99.5\%}(\chi)}{\text{BEL}_0 - 1} \times \text{BEL}_0 \quad \text{with} \quad \chi = \frac{F_1 + \text{BEL}_1}{1 + R_1},
$$

where $F_1$ denotes the cash flows payable at month 1, $R_1$ is the return on assets at month 1 and $D_0$ the duration of the liability. The best estimate of the reserve at month 0 and 1 are denoted $\text{BEL}_0$ and $\text{BEL}_1$ respectively.

The variable $\chi$ can be interpreted as the economic liability according to the assets allocation, see Planchet and Thérond (2007), while

$$
\frac{\text{VaR}_{99.5\%}(\chi)}{\text{BEL}_0 - 1}
$$

represents the one-year solvency ratio without taking into account the duration of capital commitment.

The law of the variable $\chi$ can be reasonably approximated by the sum of the discounted cash flows, i.e. $\Lambda$, which is, conditionally to the disturbance, (approximatively) gaussian, see Guibert et al. (2010, Section 3.2) :

$$
f_{\Lambda}(x) = \mathbb{P}[\Lambda \leq x] = \mathbb{E}[\mathbb{P}[\Lambda \leq x|a]] \xrightarrow{n \to +\infty} \int \Phi \left( \frac{x - \mu(a)}{\sigma(a)} \right) f_a(da).
$$

In practice, we approximate this function by Monte Carlo simulations on the basis of a sample of the variable $a$ :

$$
f_{\Lambda}(x) \approx f_K(x) = \frac{1}{K} \sum_{k=1}^{K} \Phi \left( \frac{x - \mu(a_k)}{\sigma(a_k)} \right).
$$

Then, a quantile $\rho$ is derived by solving the equation $f_K(x_\rho) = \rho$ by dichotomy. This model have been used to integrate risk of a pandemic into an internal model in the solvency II framework by Planchet (2013).

The moments of $\Lambda$ are derived in expression (1). If we consider that the monthly care costs 1 and a zero discount rate, it leads, for a portfolio of $n$ LTC-claimants to

$$
\mu_a = \mathbb{E}[\Lambda|a] = n \times \mathbb{E}[\Psi|a] \quad \text{and} \quad \sigma_a = \sigma_{\Lambda|a} = \sqrt{n \times \mathbb{V}[\Psi|a]},
$$

with $\mathbb{E}[\Psi|a] \approx \sum_{t \geq 1} S_v(t|a)$ and $\mathbb{V}[\Psi|a] \approx 2 \sum_{t \geq 1} t S_v(t|a) - (S_v(t|a))^2$, following expressions (2) and (3). We can then compute the ratio between the SCR and the best estimate of the reserve as a function of the portfolio size for different ages of occurrence for the risk of systematic deviations.
4 Numerical application

We use the smoothed mortality surface obtained in Tomas and Planchet (2012) by fitting the adaptive local likelihood model with local bandwidth factors. Figure 1 presents the conditional probabilities of death \( q_u(v) \).

![Figure 1: \( q_u(v) \) obtained in Tomas and Planchet (2012, Section 3.2).](image)

We then apply model (5) and vary \( \sigma_v \) from 1 to 20\%. The remaining life expectancy varies slightly with \( \sigma_v \), being around 41.4 months when the pathology occurred at age 80. We then measure the impact of uncertainty on the expected lifetime by computing the relative difference \( \delta \) between the 95\% quantile of the simulated remaining lifetime from 5000 simulations and its expectation, equation (6). The computations are carried out with the help of the software R, R Development Core Team (2013). Figure 2 shows the impact of uncertainty on the remaining life expectancy.

![Figure 2: Relative difference \( \delta \) between 95\% quantile of the simulated remaining lifetime and its expectation.](image)
The impact of uncertainty is relatively linear on the remaining life expectancy for a given age of occurrence. The convex shape of the surface is explained by the underlying exposition as we observe most of the exposure around the age of occurrence 80, see the observed statistics of the dataset in Tomas and Planchet (2012).

In the general reform of Solvency II, the uncertainty arising from unanticipated aggregate mortality has to be covered by the SCR (partly through the risk margin). With a level of volatility $\sigma_\epsilon = 9\%$, the resulting uncertainty $\delta$ is approximatively 12% when the pathology occurred at age 80. When computing the quantile at 99.5%, the difference with the expected remaining lifetime is around 19.5%. It means that the capital required for covering the uncertainty is approximatively 19.5% of the best estimate.

In applying a reduction of 20% on the conditional probabilities of death with the same logic as the disability / morbidity shock described in the QIS5 specifications, the remaining life expectancy of an LTC-claimant when the pathology occurred at age 80 increases from 41.4 to 49.5 months, meaning a gain of 19.6%. In consequence, setting the volatility of the disturbance at 9% appears to be relatively coherent with the calibration of the standard formula as illustrated in Figure 3.

![Figure 3: QIS5 disability / morbidity shock (black dotted line) and ratio between the 99.5% quantile of the simulated remaining lifetime and its expectation (color lines).](image)

We have applied the methodology to other LTC-claimants datasets. It appears that the results are insensitive to the underlying structure of the survival law. It is explained by the fact that we are working on the core of the distribution, i.e. the general form of the survival law. In consequence, the underlying structure has no impact. It would be appropriate to vary the volatility according the age of occurrence of the pathology, as the volatility increases at its extremes. In addition, the model only assesses the risk associated with the uncertainty arising from the systematic deviations to which the risk of random fluctuations must be added.
As mentioned in Section 3.3, computing the quantiles of the distribution of $\Lambda$ gives a biased evaluation of the SCR. In the following, we use the general approximation proposed by Guibert et al. (2010) and compute the ratio between the SCR and the best estimate of the reserve as a function of the portfolio size for different ages of occurrence for the underwriting risk. The result is displayed in Figure 4 with a level of volatility $\sigma_e$ of 9% and a cost of capital $\alpha$ of 6%.

![Figure 4: Ratio between the SCR and the best estimate of the reserve as a function of the portfolio size.](image)

At age of occurrence 80, the minimal SCR is around 38% for a portfolio of LTC-claimants of infinite size, i.e. when ignoring the risk of random fluctuations. For a size of 100 LTC-claimants, the minimal SCR is 144% of the best estimate of the reserve. In addition, we observe that for a portfolio of small size, the SCR is rapidly decreasing with the age of occurrence. As well, we have applied this model to other datasets. Unlike computing the quantiles of the distribution of $\Lambda$, the results obtained with the general approximation proposed by Guibert et al. (2010) are very sensitive to the choice of the underlying survival law. It highlights the impact of the structure of the survival law on the underwriting risk, in particular the importance of the tail of distribution.

5 Conclusions

The uncertainty associated to the underlying survival law has important consequences in terms of volatility of reserves. As soon as we are able to model this uncertainty, the general framework described in Guibert et al. (2010) allows us to assess the adequacy of the standard shocks described in the QIS5 specifications with a risk analysis specific to the insurer depending on the structure of the portfolio. We can then build a stochastic model taking into account the constraint of quantifying the uncertainty in a finite horizon and the effect of the risk margin. In this framework, the use of simulations is limited and occurs independently of the portfolio size, as in a semi-analytical model, and the computation time is limited. These reflections highlight the essential assessment of uncertainty associated to the underlying survival law as the milestone for a thorough evaluation of the insurer solvency. It leads to consider the implementation of a partial internal model for the underwriting risk. In addition, the approach presented shows that the level of underwriting SCR obtained is strongly associated to the precision of the assessment of the underlying survival law and in particular to the tail of distribution.
Références


