Solvency capital for insurance company: modelling dependence using copula

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2014.8
Solvency Capital for Insurance Company: Modelling Dependence Using Copula

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April 3, 2014

Abstract

The aim of this article is to improve the internal model of the solvency 2 framework, by assessing a solvency capital for non life insurance portfolio, taking into account potential dependencies between insured risks. We used two stochastic models and a simulation technique to determine the distribution of reserve. Then we modelled the dependence using several copulas and the best one was selected using a goodness of fit test. Finally we evaluated the solvency capital in the dependent and independent case. By comparing the results, we highlighted the effect of dependence on solvency capital of the insurance company.

Keywords: Solvency Capital Requirement, Reserves, Claims dependence, Copulas models, Simulation methods.

JEL Classification Codes: C15, G22, G17

1 Introduction

Much actuarial research in recent years has focused on the solvency of the insurance companies. Indeed, insurance company must have a level of liability (equities and technical reserve), which allows it to be solvent in future years. Historically, the insurance companies were sufficiently capitalized compared to their engagements: the markets were controlled and less volatile. Furthermore the correlation between the risks of insurance was not considered. Recently, we remark that the claims increased and the legal environment became more uncertain: the Lothar storm in 1999, and the disaster of the World

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Trade Center in 2001, were responsible for an explosion of the number of insolvency, and led to an exceptional conjunction of the disasters within the most various lines of business: damage, catastrophes, industrial accidents, trading losses, civil responsibility. Also, some situations are observed almost every day, as for example, in auto insurance, accidents may involve several insured at once in a collision. These events proved that the risks of insurance could be dependent, and this dependence can bind liabilities or assets of insurance. Thus, control of risk has become essential.

One way is to consider dependence between insured risks. For example, linear correlation is frequently used in practice as a measure of dependence notably by the solvency 2 framework see Commission (2010). However, it can not capture the non-linear dependence relationships that exist between different risks, especially when extreme events are highly dependent between lines of business.

Recently, Several actuarial studies have applied copulas functions to fully capture a wide range of dependence structure amongst different insured risks. Frees and Valdez (1998) have proposed copula function to measure dependence between risks of insurance, and to evaluate the loss of life mortality, the loss of adjustment expenses and reinsurance contract pricing. Moreover Frees and Wang (2006) have used copulas for estimating the credibility of aggregate loss. In addition, Kaishev and Dimitrova (2006) have shown the importance of copulas in reinsurance. Antonin and Benjamin (2001) also used copulas to assess the amount of the reserve, they provided a model which combined at the same time the theory of copula and the theory of credibility in order to detect better the dependence between the lines of business. Furthermore, Belgue and Levi (2002), Faivre (2002) and Cadoux and Loizeau (2004) have shown that the model with copulas allows for an aggregation of risks and evaluates a capital higher than when assuming independence. Similarly, Krauth (2007) has modeled the dependence using three models (bootstrap, common shock and copulas) to assess the amount of insurance reserves. In addition, Bargés et al. (2009) evaluated the capital allocation for the overall portfolio using the TVaR as a measure of risk and a FGM copula.

More recently Zhao and Zhou (2010) have applied semi parametric copula models to individual level insurance claims data to forecast loss reserves, and Shi and Frees (2011) have investigated the aggregate insurance loss reserving data with bivariate copulas and linear models. Besides, Diers et al. (2012) have shown the flexibility of a Bernstein copula to model a several lines of business.

All studies mentioned above have considered copulas functions as a powerful tool to resolve the problem of dependence between insurance risks. However, the new solvency 2 framework launched in 2001 which is the basis framework for the European insurance market consider the correlation coefficient to measure dependence and to aggregate risks. Or this measure is insufficient to capture the non linear dependence of insurance risks. For that, The purpose of this work consists, to model the structure of dependence between underwriting risks of non life portfolio of a Tunisian insurance company, and to assess the solvency capital requirement (SCR) by the new solvency 2 approach. Indeed this paper is primarily motivated to show the importance of the internal model of the Solvency 2 framework compared to the standard model, and to extend the internal model by including copula approach in assessing reserves and capital.
On the basis of this framework, we consider in this paper, two models, the standard model and the internal model of Solvency 2, the last one is investigated in two cases; the independence and the dependence case in which a copula approach is used to model the structure of dependence. A goodness of fit test for copula was implemented to select the best one for the data and a simulation method of conditional distribution of the copula is applied to include dependence in the internal model.

The paper is organized as follows. Section 2 presents the Solvency 2 framework. In Section 3 we present reserving models. In section 4 we provide the copulas functions. Section 5 reports the empirical results. Some comparisons with other papers are discussed in section 6 followed by concluding remarks and some open questions in Section 7.

2 Solvency 2 framework

Solvency 2 provide a practical tools to evaluate the Solvency Capital requirement (SCR) for insurance companies in order to manage their risks ( reserve risk, premium risk, catastrophe risk...). Two models were proposed: the standard model and the internal model. These models take the advantage of evaluating liabilities in stochastic way, contrary to Solvency 1, which evaluate liabilities with deterministic methods.

In this paper, we focus on reserve risk, which results from fluctuations in the timing and amount of claims payment. We present here models of SCR for reserve risk, for two lines of business denoted by \( k \) and \( l \), but the models can be generalized for more lines of business.

**Standard model**

The European Insurance and Occupational Pension Authority (EIOPA) see [Commission (2010)](https://doi.org/10.1007/s10001-010-0026-y) defines a common standard formula for all the insurance companies in order to manage their risks.

The SCR is defined as the following

\[
SCR_{aggregate} = \rho(\sigma_{total}) V_{total}
\]

where

\[
\rho(\sigma_{total}) = \frac{\exp(N_{0.995} \times \sqrt{\log(\sigma_{total}^2 + 1)})}{\sqrt{\sigma_{total}^2 + 1}} - 1
\]

\( N_{0.995} \) is the quantile of the standard normal distribution.

\[
\sigma_{total} = \sqrt{\frac{1}{V_{total}} \cdot \sum_{k,l=1}^{n} Corr_{k,l} \cdot \sigma_k \cdot \sigma_l \cdot V_k \cdot V_l},
\]

is the overall standard deviation of the insurance portfolio, and \( \sigma_k, \sigma_l \) are the standard deviations of reserve risk respectively, for the line of business \( k \) and the line of business \( l \) fixed by the EIOPA see Appendix A.

\( V_{total} = V_k + V_l \) is the total best estimate of technical reserve, which is the amount of claims would rationally pay to settle the obligations, and \( V_k, V_l \) are respectively the best estimate of reserve for the line of business \( k \) and the line of business \( l \).
**Internal model**

The EIOPA recommends the use of an internal model adapted to the risks which are really supported by the insurer. The formula of capital of reserve risk for line business $k$, is given by

$$SCR_k = VaR_{99.5\%}^k - BE_k$$

where $VaR_{99.5\%}^k$ is the Value at Risk of the line business $k$ at confidence level 99.5%. $BE_k$ is the best estimate of the total reserves of the line of business $k$. This best estimate must be calculated by at least two models using actuarial techniques (see section 3).

After computing an SCR for each line of business, we evaluate the aggregate SCR obtained by

$$SCR_{aggregate} = \sqrt{\sum_{k,l} Corr_{k,l} SCR_k SCR_l}$$

where $Corr_{k,l}$ is the correlation matrix of the two lines of business. $SCR_k, SCR_l$ are the solvency capital requirement respectively, for the line of business $k$ and the line of business $l$.

3 Best Estimate Models

We present here two models for evaluating Best Estimate of reserve risk; the Stochastic Chain Ladder models and the Generalized Linear Models.

3.1 Stochastic Chain Ladder model

Chain Ladder model is the traditional and the straightforward technique to estimate future claims amounts see [Mack (1993)] and [Mack (1994)]. This technique is applied to cumulative claims data. We assume that the data consist of a triangle of incremental claims, $\{y_{i,j}: i = 1, ..., n; j = 1, ..., n - i + 1\}$.

$i$: refers to the row, and indicate the accident year.

$j$: refers to the column and indicate the delay year (or development year).

We assume that these incremental claims are independent. Then, the cumulative claims data are obtained by

$$C_{i,j} = y_{i,j} + \sum_{k=1}^{j-1} y_{i,k}$$

The Chain Ladder methods consist of forecasting the future claims $C_{i,j}$, by estimating a development factors as

$$\lambda_j = \frac{\sum_{i=1}^{n-j} C_{i,j+1}}{\sum_{i=1}^{n-j} C_{i,j}} \quad 1 \leq j \leq n - 1$$
Then, the conditional expected loss, which is the best estimate of the future claims is

\[ E(C_{i,j+1} \mid C_{i,1},\ldots,C_{i,j}) = \lambda_j C_{i,j} \] (6)

For \( 1 \leq i \leq n \) and \( 1 \leq j \leq n \), we have \( V(C_{i,j+1} \mid C_{i,1},\ldots,C_{i,j}) = C_{ij}\sigma_j^2 \), with \( \sigma_j \) is the volatility of the development year defined as

\[ \sigma_j^2 = \frac{1}{n-j-1} \sum_{i=1}^{n-j} C_{i,j} \left( \frac{C_{i,j+1}}{C_{i,j}} - \lambda_j \right)^2 \] (7)

The amount of the expected reserve \( E(R_i = C_{i,n} - C_{i,n-i+1} \mid C_{i,1},\ldots,C_{i,j}) \) is estimated by

\[ R_i = C_{i,n} - C_{i,n-i+1} \] (8)

Thus the best estimate of the total amount of reserves is \( R = \sum_{i=1}^{n} R_i \).

The uncertainty on the estimation of reserves can be measured by

\[mse(R_i) = C_{i,n}^2 \sum_{k=n+1-i}^{n-1} \frac{\sigma_k^2}{\lambda_k^2} \left( \frac{1}{C_{i,k}} + \frac{1}{n-k} \sum_{j=1}^{n-k} C_{j,k} \right) \] (9)

In alternative measure of uncertainty is the standard error \( se(R_i) = \sqrt{mse(R_i)} \) and the coefficient of variation \( cv = \frac{se}{E(R_i)} \).

Chain Ladder is considered as the standard reserving method for reserve estimation. However, this method present several disadvantages. In fact, it assumes a linear relationship in claims between the development years; it multiply a claim in development year \( j \) by a factor \( \lambda_j \) to obtain a future claim in development year \( j+1 \), so the risk of accumulated errors is important. Also, this method does not take into account the various change in the inflation, law...

### 3.2 Generalized Linear Model (GLM)

In alternative model to the Stochastic Chain Ladder model was proposed by Nelder and Wedderburn (1972), the Generalized Linear Model (GLM). Contrary to Chain Ladder technique, this method is applied to incremental data \( y_{i,j} \).

We assume that the response variable \( y_{i,j} \) are independent and belong to the exponential family distributions defined as

\[ f(y_{i,j}, \theta_{i,j}, \phi) = \exp\left(\frac{y_{i,j}\theta_{i,j} - b(\theta_{i,j})}{a_{i,j}(\phi)} + c(y_{i,j}, \phi)\right) \] (10)

where \( a_{i,j}(\phi), b(\theta_{i,j}) \) and \( c(y_{i,j}, \phi) \) are functions specified in advance, \( \theta_{i,j} \) is a parameter related to the mean and \( \phi \) scale parameter related to the variance. for more details we refer the readers to McCullagh and Nelder (1989).
We denote the best estimate of claims amounts is $\mu_{i,j} = E(y_{i,j}) = b^{-1}(\theta_{i,j})$ and $v(y_{i,j}) = \phi v(\mu_{i,j}) = \phi \hat{b}(\theta_{i,j})$

The mean $\mu_{i,j}$ is related to the covariates via the link function $g$, that is differentiable and monotonic, such that $g(\mu_{i,j}) = \eta_{i,j}$, this link function can be: identity, log, logit and reciprocal. Alternatively, as pointed out by Merz and Wuthrich (2008), a log link is typically a natural choice in the insurance reserving context.

$\eta_{i,j}$ is the linear predictor and is defined by $\eta_{i,j} = X_{i,j}\beta$, where $X_{i,j}$ is a matrix of covariates and $\beta$ is the vector of parameters. In our context, we use accident years and development years for covariates of future claims.

The model parameters are estimated using maximum likelihood, and the best estimate is given by $E(y_{i,j}) = \hat{\mu}_{i,j} = g^{-1}(\eta_{i,j})$. The amount of reserve for each year is calculated by $R_i = \sum_{j>n-i+1} \hat{\mu}_{i,j}$ and the best estimate of the total amount of reserves is $R = \sum_{i=1}^{n} R_i$.

In order to estimate the mean squared error that measure the uncertainty of reserves, we used the Delta methods of England and Verrall (1999) that requires the variance-covariance matrix, so the mse of each year of occurrence is calculated as follow

$$mse(\hat{R}_i) = \sum_{j>n+1-i} \phi \hat{\mu}_{i,j} + \sum_{j>n+1-i} \hat{\mu}_{i,j}^2 v(\hat{\eta}_{i,j}) + 2 \sum_{j,k>n+1-i,j<k} \hat{\mu}_{i,j} \hat{\mu}_{i,k} \text{cov}(\hat{\eta}_{i,j}, \hat{\eta}_{i,k}) \quad 1 \leq i \leq n$$

and the mse of the total reserves is

$$mse(\hat{R}) = \sum_{j>n+1-i} v(y_{i,j}) + \hat{\mu}_{i,j}^2 v(\hat{\eta}_{i,j}) + 2 \sum_{j,k>n+1-i,j<k} \hat{\mu}_{i,j} \hat{\mu}_{i,k} \text{cov}(\hat{\eta}_{i,j}, \hat{\eta}_{i,k})$$

The main problem with models like those above, that Chain ladder and GLM do not take into account the dependence structure between the different lines of business that is often induced by such factors: inflation, catastrophe events. Also, another example: a car accident can cause both a material damage related to the car and a human damage related to the driver. Another example for dependence is the climate states (snowy, rainy) can influence the number of car accident as their amounts of claims. In addition these models can’t predict an accurate reserves since the data are non linear and non Gaussian. For that, a new type of model must be introduced to fill these gaps.

In the next section we introduce the copula function that has been discovered for creating bivariate distributions and taking into account the dependence between marginals.

### 4 Copulas

In this section, we provide an appropriate statistical model for dependent claims amounts relative to two lines of business. Copulas functions have been introduced in the insurance context by Frees and Valdez (1998). A copula is based on an assumption that both marginal distributions are known.

Let $F_X(x)$ and $F_Y(y)$ denote the marginal distribution functions of the variables $X$ corresponding to the claims amounts of the line Auto Damage and $Y$ corresponding to the claims amounts of the line Auto Liability.
The joint distribution function $F_{X,Y}(x,y)$ is then obtained as

$$F_{X,Y}(x,y) = C[F_X(x), F_Y(y)]$$

where $C(u,v)$ is the copula, a cumulative distribution function for a bivariate distribution with support on the unit square and uniform marginals.

In this paper, we assume that the marginal distributions are continuous with density functions $f_X(x)$ and $f_Y(y)$. Then, the joint density function is

$$f_{X,Y}(x,y) = f_X(x)f_Y(y)C_{12}[f_X(x), f_Y(y)]$$

where

$$C_{12}(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v}$$

The conditional distribution function of $Y \mid X = x$ is

$$F_{Y\mid X}(y \mid x) = C_1[F_X(x), F_Y(y)]$$

where

$$C_1(u,v) = \frac{\partial C(u,v)}{\partial u}$$

In the insurance context, Frees and Valdez (1998) provide a number of copulas. In each case, the parameter $\alpha$ measures the degree of association. In the present work, we used several copulas in order to select the appropriate one, that describe the structure of dependence between the two lines of business.

We investigate the Archimedean copulas and their survival copulas, which are more appropriate to insurance data than elliptical copulas, because elliptical copulas are generally applied to symmetric distributions.

**Gumbel**

This copula models a positive dependence and represents the risks which are more concentrated in the upper tail. It is defined as

$$C(u,v) = \exp\{-[(-lnu)^{\alpha} + (-lnv)^{\alpha}]^{1/\alpha}\} \quad \alpha \geq 1$$

where $\alpha$ is the parameter dependence. The Gumbel copula has upper tail dependence $\lambda_U = 2 - 2^{1/\alpha}$, but no lower tail dependence.

**Fank**

This copula models both positive and negative dependence, but it has no tail dependence, it is defined as

$$C(u,v) = -\frac{1}{\alpha}ln[1 + \frac{(e^{-\alpha u} - 1)(e^{-\alpha v} - 1)}{(e^{-\alpha} - 1)}] \quad \alpha \neq 0$$
**Clayton**
This copula models positive dependence. Unlike to the Gumbel copula, it represents the risks which are more concentrated in the lower tail, so it correlates small losses. It is defined as
\[
C(u, v) = (u^{-\alpha} + v^{-\alpha} - 1)^{-1/\alpha} \quad \alpha \in [-1, \infty) \setminus \{0\}
\]
The Clayton copula has lower tail dependence \( \lambda_L = 2^{-1/\alpha} \) and \( \lambda_U = 0 \).

**Rotated Gumbel**
It is the survival copula of the Gumbel copula, it is defined by
\[
C(u, v) = u + v - 1 + C_G(1 - u, 1 - v; \alpha) \quad \alpha \in [1, \infty)
\]
where \( C_G \) is the Gumbel copula.
The Rotated Gumbel copula has a lower tail dependence \( \lambda_L = 2 - 2^{1/\alpha} \) and \( \lambda_U = 0 \).

**Rotated Clayton**
It is the survival copula of the Clayton copula, it is defined by
\[
C(u, v) = u + v - 1 + C_C(1 - u, 1 - v; \alpha) \quad \alpha \in [-1, \infty) \setminus \{0\}
\]
where \( C_C \) is the Clayton copula.
The Rotated Clayton copula has an upper tail dependence \( \lambda_U = 2^{-1/\alpha} \) and \( \lambda_L = 0 \).

By using these copulas, we are able to detect dependence in the tails.

In order to estimate copulas model, we are based on the Canonical maximum likelihood method. This method consists in transforming the data of claims amount \( (x_{1t}, \ldots, x_{Nt}) \) into uniform variates \( (\hat{u}_{1t}, \ldots, \hat{u}_{Nt}) \) using the empirical distribution functions, and then estimate the parameter in the following way
\[
\hat{\alpha} = \arg\max \sum_{t=1}^{T} \ln c(\hat{u}_{1t}, \ldots, \hat{u}_{Nt}; \alpha)
\]
In the next section, we report empirical results obtained from using the models above.

5  **Results**

5.1  **Modeling SCR with the Solvency 2 framework**

Data were supplied by a Tunisian insurance company and consist of claims amounts of two lines of business Auto Damage and Auto Liability settled at a corresponding accident year and development year. These claims amounts are net of reinsurance, classified in so called run off triangle. Here, \( n = 8 \) accident years are available (from 2001 to 2008). The line Auto Damage corresponds to the damage of the car. It is a short line of business, that once the claim is declared it will be paid after a short time.
The line Auto liability is defined as a legal obligation for anyone to repair the damage caused to others, it is called a slow development line of business, because there will be a time lag between the date of occurrence claim and the date of full refund.

In order to estimate the future claims amounts and the reserves, we begin by selecting the adequate parametric distribution for the data. We carried out a Kolmogorov-Smirnov goodness of fit test for the two lines of business. This test assess the relationship between the empirical distribution and the estimated parametric distribution. A large p-value indicates a non significant difference between the two. We tested three distributions: Poisson, Log normal and Gamma, that belong to the exponential family, and have been extensively studied for incremental claims amounts in the loss reserving literature, see England and Verrall (1999). As can be seen from Table 1, the p-value of the Kolmogorov-Smirnov test indicate that the Gamma model is the better fit for the data. Claims amounts of the two lines of business follow approximately the Gamma distribution. This is consistent with actuarial research. In fact, this distribution is well known in actuarial modelling of claims distribution (see England and Verrall (1999) and Mack (1991)).

For this data set two models are fitted, the Stochastic Chain Ladder and the Generalized Linear Model. The estimated total reserves with their relative standard errors and coefficient of variations are listed in Table 2.

Table 1: Kolmogorov-Smirnov test

<table>
<thead>
<tr>
<th></th>
<th>Poisson</th>
<th>Log Normal</th>
<th>Gamma</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-value Auto Damage</td>
<td>$1.3710^{-5}$</td>
<td>0.04</td>
<td>0.1</td>
</tr>
<tr>
<td>P-value Auto Liability</td>
<td>$1.5110^{-5}$</td>
<td>0.03</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 2: Total reserves (in thousands TND), standard errors and coefficient of variations for the lines of business Auto Damage and Auto Liability

<table>
<thead>
<tr>
<th></th>
<th>Auto Damage</th>
<th>Auto Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stochastic Chain Ladder</td>
<td>51.037</td>
<td>177.155</td>
</tr>
<tr>
<td>GLM Gamma</td>
<td>45.343</td>
<td>179.251</td>
</tr>
<tr>
<td>cv (%)</td>
<td>22.02</td>
<td>28.5</td>
</tr>
<tr>
<td>cv (%)</td>
<td>21.69</td>
<td>27</td>
</tr>
</tbody>
</table>

Although the solvency 2 framework requires the use of different actuarial techniques to evaluate the total reserves, one can conclude from results that the reserve estimates for each lines of business with Stochastic Chain Ladder and GLM Gamma are quite comparable, with some small differences. Looking for the Auto Damage, the GLM Gamma provide a total reserves lower than the Stochastic Chain Ladder and inversely for the Auto Liability, it provide a total amount of reserves slightly more than Stochastic Chain Ladder model. Auto Liability is considered as a slow development line of business, so GLM Gamma estimate a prudent amount of reserve.
For the measure of uncertainty of reserve, Standard errors and the coefficient of variations of GLM Gamma are lower than Stochastic Chain Ladder, so GLM model is considerably more robust, and can estimate the accurate reserves for the claims. We retain this model to evaluate Value at Risk and SCR.

In order to evaluate Value at Risk of reserves, we need to determine the whole distribution for the total reserves. For that we use the bootstrap approach. In the following we present the algorithm of bootstrap process.

First, we fit the data and estimate the best estimate of claims amounts $\hat{\mu}_{i,j}$ of the run-of triangle using GLM Gamma.

Second, we calculate the residuals off triangle as follows $\frac{y_{i,j} - \hat{\mu}_{i,j}}{\sqrt{V(\hat{\mu}_{i,j})}}$.

Third, we resample residuals 10000 times. A bootstrap data sample is then created by inverting the formula for the residuals using the resampled residuals, together with the fitted best estimate of claims amounts $y_{i,j} = \hat{r}_{i,j} \ast \sqrt{V(\hat{\mu}_{i,j})} + \hat{\mu}_{i,j}$.

Having obtained the bootstrap sample, here 10000 run off triangle, the GLM Gamma model is refitted and we calculate reserves of accident years, and the total reserves for each triangle. Finally we obtain the distributions of $R_i$ for each accident years and the whole distribution of the total reserves. (See appendix A for the distribution of total reserves of the two lines of business).

Having the distributions of total reserves, Value at risk is calculated by applying the empirical VaR (quantile) at a confidence level 99.5%, and best estimate (BE) is calculated as the mean of the total reserves.

**Table 3: VaR$_{99.5\%}$ (in thousands TND) for the Auto Damage and Auto Liability**

<table>
<thead>
<tr>
<th>Year</th>
<th>VaR$_{99.5%}$ Auto Damage</th>
<th>VaR$_{99.5%}$ of Auto Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2002</td>
<td>1.604</td>
<td>1.997</td>
</tr>
<tr>
<td>2003</td>
<td>2.286</td>
<td>3.550</td>
</tr>
<tr>
<td>2004</td>
<td>3.836</td>
<td>7.061</td>
</tr>
<tr>
<td>2006</td>
<td>14.215</td>
<td>37.655</td>
</tr>
<tr>
<td>2007</td>
<td>24.146</td>
<td>103.715</td>
</tr>
<tr>
<td>2008</td>
<td>65.168</td>
<td>292.528</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>122.464</strong></td>
<td><strong>474.510</strong></td>
</tr>
<tr>
<td><strong>Aggregate</strong></td>
<td><strong>87.447</strong></td>
<td><strong>383.869</strong></td>
</tr>
</tbody>
</table>

**Table 4: SCR Standard Model versus SCR Internal Model (in thousand TND)**

<table>
<thead>
<tr>
<th></th>
<th>SCR Standard Model</th>
<th>SCR Internal Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auto Damage</td>
<td>10.198</td>
<td>36.457</td>
</tr>
<tr>
<td>Auto Liability</td>
<td>65.051</td>
<td>203.171</td>
</tr>
<tr>
<td><strong>SCR$_{Total}$</strong></td>
<td>75.249</td>
<td>239.629</td>
</tr>
<tr>
<td><strong>SCR$_{Aggregate}$</strong></td>
<td>69.506</td>
<td>61.51</td>
</tr>
</tbody>
</table>
It is noticed from the results of Tale 3 that the VaR\textsubscript{99.5\%} increases remarkably over the accident years. As these recent years are more uncertain, they require a high level of reserves. Moreover, the VaR\textsubscript{99.5\%} of aggregate reserves is always less than the sum of VaR\textsubscript{99.5\%} of each accident year. That means, it should more capital to cover independently each risk, than to cover the aggregate risk. In this configuration, the diversification between accident years has a significant influence on the calculation of reserves which leads to improve the solvency and the efficiency in the insurance company. So VaR\textsubscript{99.5\%} of aggregate reserves was used to evaluate the SCR.

From Table 4, we remark that the aggregate SCR evaluated with the standard or internal model is always lower than that in total. Specifically, for the internal model the difference is outstanding, the diversification is more remarkable than in the standard model. Thus the diversification between lines of business in insurance reduces the overall risk and provide a solvency capital which takes into account the compensation between risks. This can be explained by the fact, that the internal model is based on a quantification of internal and proper risks of the company. Unlike the Standard model that uses statistics established by the EIOPA (see Appendix A and Commission (2010)). Finally, it appears that the evaluation of the SCR with the internal model seems to be the most interesting; it calculates a coverage capital in adequacy with the proper risks of the company, ensuring both solvency for the insured and the economy for equities.

5.2 Modeling SCR with the Copula approach

As mentioned in section 2 the aggregation of risks in the Solvency 2 approach is done using the correlation coefficient, however, the dependence is in general non linear and the risks are non-Gaussian. So, we introduced in our paper the copulas function to model the dependence between the Auto Damage and Auto Liability. In order to estimate the parameter of dependence \( \alpha \), we fit the claims amounts relative to the two lines of business by maximizing the log likelihood of each copula function. We present here the results obtained from these models.

<table>
<thead>
<tr>
<th>Copula</th>
<th>LL</th>
<th>( \alpha )</th>
<th>AIC</th>
<th>BIC</th>
<th>( \lambda_U )</th>
<th>( \lambda_L )</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gumbel</td>
<td>2.02</td>
<td>1.35</td>
<td>-3.98</td>
<td>-3.94</td>
<td>0.33</td>
<td>0</td>
<td>0.038</td>
</tr>
<tr>
<td>Frank</td>
<td>4.78</td>
<td>3.37</td>
<td>-9.49</td>
<td>-9.45</td>
<td>0</td>
<td>0</td>
<td>0.21</td>
</tr>
<tr>
<td>Clayton</td>
<td>8.56</td>
<td>1.32</td>
<td>-17.07</td>
<td>-17.03</td>
<td>0</td>
<td>0.59</td>
<td>0.27</td>
</tr>
<tr>
<td>Rotated Gumbel</td>
<td>6.7</td>
<td>1.62</td>
<td>-13.34</td>
<td>-13.3</td>
<td>0</td>
<td>0.46</td>
<td>0.017</td>
</tr>
<tr>
<td>Rotated Clayton</td>
<td>0.96</td>
<td>0.42</td>
<td>-1.86</td>
<td>-1.81</td>
<td>0.19</td>
<td>0</td>
<td>0.01</td>
</tr>
</tbody>
</table>

It is noticed from the results of Table 5 that the Clayton copula is the more adequate for modeling dependence of the two lines of business Auto Damage and Auto Liability. This copula presents the maximum log-likelihood and the minimum information criteria AIC and BIC. It capture a positive association, \( \alpha = 1.32 \). This can be interpreted as, when the claims amounts of the line of business Auto Damage are high (respectively low),
those of the line of business Auto Liability have more risk to be high (respectively low).

As can be seen, results of tails dependence of the Clayton copula show that both insured risk are independent in the up level of claims amounts distribution and dependent on the lower level of the latter. In other words, in car accident, there is a non null probability to have a risk in the line Auto Liability knowing that already has a risk in the line Auto Damage. Thus, the Clayton copula is the best candidate to model dependence between the two triangles of claims.

In addition, to validate our choice we proceeded to implement a goodness of fit test of copulas: the Kendall’s process test (for more details about this test see [Genest et al., 2009]). We remark that this copula present the highest p-value of the Kendall’s process test. So it is more appropriate to describe the structure of dependence of the two lines of business. Consequently, we retain this copula to assess the solvency capital.

In order to evaluate the SCR of the internal model (2), we carried out a simulation method in two cases: dependent case and independent case.

**Dependent Case**

To simulate the joint distribution via the subjacent copula (in our case the Clayton copula) and obtain estimates for the $VaR_{99.5\%}$, we used the conditional sampling method [1] as follows.

- Generate two independent uniform random vectors, $u$ and $v$.

- Let the conditional distribution of $v$ given $u$ be

  $$ p = C(v | u) = \frac{\partial C(u, v)}{\partial u} = v^{-\alpha - 1}(u^{-\alpha} + v^{-\alpha} - 1)^{-\alpha - 1} $$

  \hspace{1cm} (20)

- Solve the above expression to obtain $v$, a function of the conditional probability $p$ and $u$, so

  $$ v = \left[ \frac{u^{-\alpha} - 1}{p^{-\alpha + 1} - 1} \right]^{\frac{1}{\alpha}} $$

  \hspace{1cm} (21)

- Transform the uniform random vectors $(u, v)$ into the vectors $(y_{i,j}^1, y_{i,j}^2)$, using the inverse Gamma function, $y_{i,j}^1 = F^{-1}_1(u)$ and $y_{i,j}^2 = F^{-1}_2(v)$, where $F_1$ and $F_2$ are the estimated marginal distributions of the Auto Damage and Auto Liability.

The total future claims amounts, $y_{i,j}^{total} = y_{i,j}^1 + y_{i,j}^2$ and the total amount of reserve, $R_{dep} = \sum_{i+j\geq n} y_{i,j}^{total}$.

By repeating these steps 10000 times, we obtain the distribution of the total reserves on which the $VaR_{99.5\%}$ and the $BE$ (which is the mean of the total reserves) are calculated.

---

1 For more details about generation of random vectors via copulas, see [Nelsen, 2007] and [Cherubini et al., 2004].
Independent Case

- Generate two independent uniform random vectors, $u$ and $v$.
- Transform the uniform random vectors $(u, v)$ into the vectors $(y_{i,j}^{1}, y_{i,j}^{2})$, using the inverse Gamma function, $y_{i,j}^{1} = F_{1}^{-1}(u)$ and $y_{i,j}^{2} = F_{2}^{-1}(v)$.

The total future claims amounts, $y_{i,j}^{total} = y_{i,j}^{1} + y_{i,j}^{2}$ and the total amount of reserves, $R_{indep} = \sum_{i+j \geq n} y_{i,j}^{total}$.

Like in the dependent case, we repeated these steps 10000 times to obtain the distribution of the total reserves on which the SCR of the internal model is calculated.

5.3 Impact of dependence on the internal model of Solvency 2 framework

In this section, we analyse the impact of dependence on the evaluation of reserves and SCR. A striking illustration of the impact of dependence on the total reserves can be seen in Fig 1 and Fig 2.

Figure 1: Scatter plots of the total reserves in independent case (in the left) versus dependent case (in the right)

![Figure 1: Scatter plots of the total reserves in independent case (in the left) versus dependent case (in the right)](image)

Figure 2: Distribution functions of the total reserves independent case (on top) versus dependent case (below)

![Figure 2: Distribution functions of the total reserves independent case (on top) versus dependent case (below)](image)
Comparing the two plots of Fig 1 shows that the scatter plot of the total reserves evaluated in dependent case is considerably more extended than reserves of the independent case. Also, Fig 2 display that distribution function of total reserves in dependent case is below that corresponding to the independent case. Indeed, for a given level of probability, taking into account the dependence leads us to put amount of reserve higher than when we assume independence between lines of business. Afterwards, we present numerical results of modelling SCR with internal model, results are summarized in Table 6.

Table 6: VaR\(_{99.5\%}\), Standards errors, Coefficients of variation and SCR in dependent case vs independent case

<table>
<thead>
<tr>
<th></th>
<th>Dependent case</th>
<th>Independent case</th>
</tr>
</thead>
<tbody>
<tr>
<td>VaR(_{99.5%})</td>
<td>360.970</td>
<td>335.710</td>
</tr>
<tr>
<td>se</td>
<td>16.824</td>
<td>15.095</td>
</tr>
<tr>
<td>cv%</td>
<td>7.5</td>
<td>6.7</td>
</tr>
<tr>
<td>SCR</td>
<td>136.375</td>
<td>111.115</td>
</tr>
<tr>
<td>Deviation/ Independent</td>
<td>22.7%</td>
<td></td>
</tr>
</tbody>
</table>

From Table 6, we remark that the model with the Clayton Copula increases the uncertainty (se) and therefore increases the level of reserves. We note that the VaR\(_{99.5\%}\) in the dependent case (model with the Clayton copula) is higher than the independent model, this is explained by the fact that taking into account the dependence between the two lines of business leads to estimate a prudent level of reserve. As we can see, the solvency capital computed in the dependent case is much higher than what was estimated in the independent case. In fact, a car accident can affect the line Auto Damage as well as the line Auto Liability, as it causes a material damage, and it can frequently cause physical injuries related to the civil responsibility. In addition, the deviation between independent and dependant SCR is of 22.7%. So the assumption of independence, can generate a risk of error in estimating the SCR of 22.7%. Also, in order to highlight the phenomenon of dependence, we compare results of Table 6, with Table 4 we can show that the SCR of the internal model evaluated with the copula approach is much higher than that assessed with the internal model of Solvency 2 as well as the aggregate SCR evaluated with the standard model. We can say that the estimate of this dependence is qualified as additional information that has in December 31, 2008 for the eight future years in order to immobilize an accurate solvency capital.

6 Discussion

Our study is within the scope of future solvency 2 framework. Indeed, the Solvency Capital Requirement is evaluated based on models proposed by the EIOPA. However actuarial studies in literature used risk measures (VaR, TVaR) at different thresholds. Cadoux and Loizeau (2004) used a portfolio of monthly data, they modelled the dependencies of intra lines of business using the Gumbel and Frank copulas that respectively
model in intensive and symmetrical dependence. They relied on a khi-deu test to select the best copula. Nevertheless, this test requires a division into classes, which reduces its power. They showed that the Value at Risk increases of 33% to 41% from the VaR in the independence case. This results is consistent with our study which is about 22.7%.

Krauth (2007) showed that the effect of dependence between lines of business occurs only at the extreme quantiles. This study shares a common assumption with us since, she modelled the distributions of lines of business using Gamma distribution and fitted the claims amounts with two models Chain Ladder and GLM. Her results are similar to us.

Antonin and Benjamin (2001) detected a positive dependence and retained the Correlated Frailty Copula, which models a strong dependence in the upper tail. Conversely, to our finding that our lines of business display a lower tail dependence. This difference is due to the characteristics of lines of business that represent in intensive dependence between the high claims amounts.

Also, they have criticized the copula model which tends to impose the same copula for each cell of triangle. Therefore, they proposed a model that combines the credibility theory and the copula approach, which was not possible in our study, since the model requires additional information for each line of business.

An alternative approach is presented by Shi and Frees (2011), where a copula regression model was used to model the dependence between two lines of business. This study exhibit a negative association between a personal and a commercial auto lines. They showed that if two subportfolios are negatively associated, one expects to see a predictive distribution that is tighter than the product copula (independence case). Contrary to our study that exhibit a positive dependence and the predictive distribution (estimated in the dependence case) spreads out more than in the independence case.

7 Conclusion

Control of risk has become a major issue in insurance where the risks are more dangerous and more concentrated. Previous research studied reserves and capital based on the independence assumption between lines of business. However, claims prove that they are dependent. It is within the framework of assessing a solvency capital requirement and the search for a model integrating the dependence may exist between lines of business in insurance, which are the objectives of our work.

We carried out an analysis of a non life portfolio consists of two lines of business Auto Damage and Auto Liability. We assessed the SCR by the two models proposed by Solvency 2 and we noticed that the aggregate SCR of the internal model is lower than the SCR in total, and this is due to diversification effect and compensation between risks. This result illustrates a potential benefits for in insurance company since, it reduces the cost of capital. An important implication of this observation is that the insurer might consider expanding the Auto Damage or shrinking the Auto Liability to take best advantage of the diversification effect. Thus, we retained the internal model that provides a capital an adequacy of the proper risks of the company. The main contribution in this paper is to improve the internal model chosen. As innovative; we propose to introduce the copula
approach in the internal model. The analysis between the two lines of business revealed a structure of dependence at the lower tail distribution described by the Clayton copula. The evaluation of the SCR showed that taking into account the dependence increases the solvency capital. So our study provides a model to assess the accurate reserve and SCR for the insurance companies that are adapted to the characteristic of lines of business and suggest that evaluating SCR with the copula approach is more appropriate.

This study is a first step of modeling dependence between two lines of business. Future work should therefore include several lines of business. Also, an important question is to determine the dependence between claims of the one run off triangle. Finally, and after assessing the SCR, it is interesting to conduct an adequate allocation of the latter to evaluate the performance of each line of business.

References


Appendix A

Table 7: Standard deviations of the lines of business Auto Damage and Auto Liability fixed by the EIOPA

<table>
<thead>
<tr>
<th>Lines of Business</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auto Damage</td>
<td>7 %</td>
</tr>
<tr>
<td>Auto Liability</td>
<td>12 %</td>
</tr>
</tbody>
</table>

Figure 3: Total reserves distributions of the lines of business Auto Damage (in the left) and Auto Liability (in the right)